AI Approaches to Cognitive Systems – The Example of Spatial Cognition

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Cognitive Systems Research

Many everyday situations we encounter are easy for most people to cope with. We can engage in communication, fill the refrigerator with our shopping items, or plan a trip to somewhere else. All this may happen effortlessly and we do not realize that these tasks involve complex cognitive activities. If, however, one tries to understand the principles behind these capabilities and tries to replicate them in computers or robots, it quickly becomes obvious that these cognitive activities are rather complex, difficult to understand, and even harder to replicate.

The interdisciplinary research area *cognitive science* is concerned with understanding the principles of cognition using psychological experiments. In addition, cognitive science researchers provide computational models that account for the experimental evidence and allow one to make predictions on cognitive behavior. In this context, timing of responses or error rates often provide helpful clues to understanding cognitive processes. In contrast to this empirical methodology, in artificial intelligence (AI) researchers typically do not try to understand the details of human cognition, but instead devise computational methods to implement certain cognitive functions on a computer, such as language understanding, route planning, or mobile robot manipulation on the basis of theoretical considerations. Although AI and cognitive science pursue different research goals, there is a large overlap in research interests and methods between the two fields, and results from one field are used in the other.

Spatial Cognition

One particular field of interest, both for AI and cognitive science, is the area of cognition that is concerned with space, *spatial cognition*. There is a great body of evidence on how humans (and animals) reason about space, how they navigate through familiar and unknown environments without getting lost, how they act in spatial environments, how they interact in space, and how they communicate spatial information. One of the major challenges for current research is to unveil the mechanisms behind these abilities and to utilize the knowledge to construct systems that assist humans in an effective manner. The starting point for much of the research in this area is the hypothesis that cognitive agents – i. e. humans, animals, robots, or computer programs – apprehend their spatial environments through (1) mental or computational operations, specifically association and reasoning; (2) perception and action in space; or (3) communication in or about space; and other forms of interaction.

In all cases, spatial structures are interpreted and computationally transformed into new structures; the new structures reflect insights about spatial situations. They form the basis for further reasoning processes, for actions in the spatial environment or in external representations (e. g., maps), and for the interaction with other agents.

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Abstract

Cognitive abilities can be studied by observing and interpreting natural systems or by developing artificial systems that interact with their environments in intelligent ways. Cognitive systems research connects both approaches. Typically, human requirements are in the focus of interest and systems are developed to interact with humans in as natural ways as possible. To achieve this goal, a deep understanding of human cognition is required. The present paper focuses on *spatial cognition*, i. e. the ability of perceiving and conceiving spatial environments and solving spatial tasks intelligently. It discusses artificial intelligence approaches to spatial cognition for supporting human activities.

Spatial structures are omnipresent in cognitive agents and around them. From a developmental point of view, the use of spatial structures is a more basic ability than abstract reasoning [26]. Therefore, the study of spatial representations and spatial computation also is very important as a basis for understanding the more general cognitive abilities of natural and artificial agents.

Spatial tasks involve a multitude of aspects, such as topological, ordinal, and metric aspects; these are manifest in different spatial reference systems and in structures formed by orientation, neighborhood, and proximity evaluation [35]. Some tasks require highly specialized competences. Frequently, however, specialized processes must be integrated to obtain powerful spatial cognition systems.

Many research endeavors in the area of spatial cognition investigate cognitive agents in spatial environments. Several projects address the question how cognitive agents can assist one another in solving spatial tasks such as reasoning about space, map comprehension, navigation, and understanding and evaluating actions in space. Other projects study how to communicate about space using language and maps to enable this assistance. The research is concerned with mental processes and structures underlying behavior in large-scale space, environmental space, vista space, and table-top space environments [22]. Solving spatial tasks in these environments requires adequate representation and processing as well as body locomotion or the movement of physical or mental objects. In addition, since most of the mentioned spatial tasks involve the change of spatial configuration, there is also a temporal dimension to spatial cognition, and there is the natural question of how spatial and temporal cognition interact.

Dimensions of Spatial Representations

When we speak about space, we refer to notions of location, orientation, shape, size (height, width, length and their combination), connection, distance, neighborhood, etc. When we speak about time, we refer to notions of duration, precedence, concurrency, simultaneity, consequence, etc. Some of the notions have well-defined meanings in disciplines like physics, topology, geometry, and theoretical computer science; but here we are concerned with the question how humans think and talk about them, how they represent such notions to get around in their spatio-temporal environment, how they reason successfully about the environment, and how they solve problems based upon this reasoning. In AI, these questions were first addressed in the framework of naive physics research [14].

There is a multitude of ways in which space and time can be conceptualized, each of which rests on implicit assumptions or explicit knowledge about the physical structure of the world. We will start with a common sense picture, which could be something like: space is "a collection of places which stand in unchanging relative position to one another and which may or may not have objects located at them"; time is "an ever-growing arrow along which changes take place." Implicit in these pictures are the assumptions that the time arrow grows even when no other changes are taking place, that space is there even when there are no objects to fill it, and that spatial relations and changes can be observed and described. As these assumptions cannot be redeemed in practice, it is more reasonable to assume that objects and events constitute space or time, respectively.

Another distinction concerns the question whether space or time should be modeled by infinite sets of (extensionless) points or by finite intervals (or regions). If we talk about Staufen being located South-West of Freiburg, it is likely that we think of two geometric points (without spatial extension) on a map of Germany. If, in a different situation, we say that you have to follow a certain road through Freiburg to reach a particular destination, Freiburg will be considered to have a spatial extension. Also, it is not clear from the outset whether a discrete, a dense, or a continuous representation of time and space may be more adequate for human cognition or for solving a given task [13]: if we want to reason about arbitrarily small changes, a dense representation seems to be a good choice; if we want to express that two objects touch each other and we do not want anything to get in between them, a discrete representation seems preferable; if on one level of consideration a touching relation and on another level arbitrarily small changes seem appropriate, yet another structure may be required. Nevertheless, a continuous structure (e. g., R^2) is often assumed which provides a better correspondence with models from physics.

Qualitative vs. Quantitative Descriptions

Space and time can be described in terms of external reference values or by reference to domain-internal entities. For external reference, usually standardized quantities with regular spacing (scales) are used; this is done particularly when precise and objective descriptions are desired; the described situations can be reconstructed accurately (within the tolerance of the granularity of the scale) in a different setting. In contrast, domain-internal entities usually do not provide regularly spaced reference values but reference values which happen to be prominent in the given domain. The internal reference values define regions which correspond to sets of quantitatively neighboring external values. The system of internally defined regions is domain-specific.

Which of the two ways of representing knowledge about a physical environment is more useful for a cognitive system? In our modern world of ever-growing standardization we have learned that common reference systems and precise quantities are extremely useful for a global exchange of information. From an external perspective, the signals generated in receptor cells of (natural and artificial) perception systems also provide quantitative information to the successive processing stages. But already in the most primitive decision stages, for example in simple threshold units, rich quantitative information is reduced to comparatively coarse qualitative information, when we consider the threshold as an internal reference value.

We can learn from these considerations, that information reduction and abstraction may be worthwhile at any level of processing. As long as we stay within a given context, the transition from quantitative to qualitative descriptions does not imply a loss of precision; it merely means focusing on situation-relevant distinctions. By using relevant entities from within a given environment for reference, we obtain a customized system that is able to capture the distinctions relevant in the given domain. Customization as an information processing strategy was considered expensive when information processing power was centralized; but with decentralized computing, as we find in biological and in advanced technical systems, locally customized information processing may simplify computation and decision-making considerably.

Significant decisions frequently are not only of local relevance; thus it must be possible to communicate them to other environments. How can we do this if we have opted for qualitative local descriptions? To answer this question, we must first decide which are the relevant aspects that have to be communicated. Do we have to communicate precise quantitative values as, for example, in international trade or do qualitative values like trends and comparisons suffice?

In cognitive systems, a qualitative description of a local decision frequently will suffice to "get the picture" of the situation; the specific quantities taken into account may have no particular meaning in another local context. Qualitative descriptions can convey comparisons from one context to another, provided that the general structures of the two contexts agree. If the descriptions refer to the spatio-temporal structures of two different environments, this will be the case.

Now consider qualitative spatio-temporal descriptions in a given environment. As they compare one entity to another entity with respect to a certain feature dimension, they form binary (or possibly higher order) relations like *John is taller than the arch* or *Ed arrived after dinner was ready*. In concrete situations in which descriptions serve to solve certain tasks, it only makes sense to compare given entities to specific other entities. For example, comparing the size of a person to the height of an arch is meaningful, as persons do want to pass through arches and comparing the arrival time of a person to the completion time of a meal may be meaningful, as the meal may have been prepared for consumption by that person; on the other hand, it may not make sense to compare the height of a person to the size of a shoe, or the arrival time of a person at home with the manufacturing date of some tooth paste. For this reason, we frequently abbreviate binary spatial or temporal relations by unary relations (leaving the reference of the comparison implicit). Thus, to a person understanding the situation context, the absolute descriptions *John is tall* and *Ed arrived late* in effect may provide the same information as the previous statements in terms of explicit comparisons.

Formal Approaches to Spatial Representation and Reasoning

In particular the idea of representing time and space in qualitative ways has been the starting point of many research endeavors in artificial intelligence. Representing spatial information and reasoning about this information is an important subproblem in many applications, such as geographical information systems (GIS), natural language understanding, robot navigation, and document interpretation. Often this information is available qualitatively, for instance when a GIS query or integrity condition has to be specified [33]. Similarly, in document interpretation, the precise size and location of layout objects is not of interest, but the relative position of these objects matters [34].

A number of approaches to representing qualitative spatial information and reasoning about space have been explored. A very early attempt at qualitative spatial representation and reasoning is Kuipers' TOUR model [19], which addresses the navigation problem using qualitative descriptions. Other approaches aim, for instance, at capturing spatial notions using first-order logic [5, 28], or even address representation and reasoning over spatio-temporal configurations [24].

All these approaches rely on quite expressive languages to talk about space. In contrast, there are approaches based on constraint satisfaction, which have a rather limited expressiveness and usually reasonably good computational properties [32]. The characteristic of these methods is that one has a system of (usually binary) relations, which is used to relate the objects of interest. For example, one can specify the relative position of layout objects using the relations *left* and *right* as well as *above* and *below*. Using this vocabulary, we can, for instance, state that an object A is *left & above* of an object B, which in turn is *right & above* of an object C. Having given these descriptions, it is obvious that the additional statement A *below* C is incompatible with what has been stated above.

Meanwhile there exist a large number of reasoning systems of this type. The first calculus in this family was Allen's *interval calculus* [1], which was originally used for reasoning about qualitative temporal information. However, this one-dimensional calculus can also be interpreted spatially [7, 16]. Furthermore, it can be generalized to two and more dimensions by projecting the objects of interest onto the axis of the coordinate system and describe the relationship between objects by the relationships between the projections [2, 12]. For example, the qualitative description of the relative position of layout objects sketched above can be done using the 2D version of Allen's interval calculus.

An interesting but less expressive approach to representing orientational relationships between extended spatial entities was introduced by Goyal and Egenhofer [11]. Their calculus consists of a 3×3 "direction-relation matrix" which represents the nine sectors formed by the minimal bounding axes of an extended spatial entity. Later, Liu et al. [21] developed reasoning algorithms for this calculus and analyzed its computational properties.

Other qualitative spatial reasoning systems are, for example, a calculus for reasoning about *orientations* [8], a calculus for reasoning about *cardinal directions* [6, 20], the *dipole calculus* [23], and a calculus for describing *2D orientations using cyclic orderings* [15].

One particular prominent reasoning system is a system of *topological relations* called *RCC-8* [29] (which is very similar to Egenhofer's 9-intersection system [4]). Given two spatial regions X and Y, there are eight possible relations between them (see Fig. 1).

All the mentioned approaches share the property that reasoning in these calculi can be done by *constraint propagation* over systems of binary (or sometimes ternary) relations with infinite domains.

In order to describe the reasoning technique that is used in most of the qualitative spatial calculi, we will use the RCC8 system. Starting with the relations of the RCC8 system, one can express that, for example, region X overlaps region Y. If we want to describe indefinite information such as the fact that $\mathrm{EC}(X, Y)$ *or* $\mathrm{PO}(X, Y)$, it is also necessary to consider the *set-theoretic unions* (corresponding to the logical

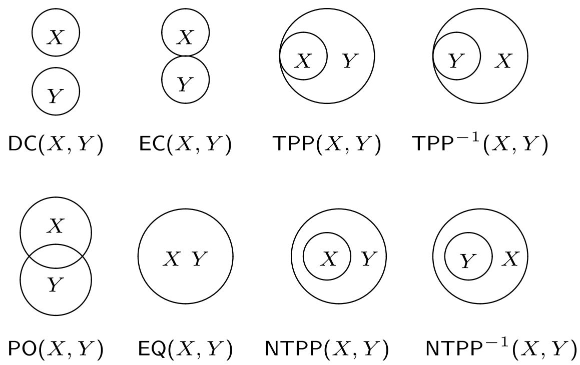

***Fig. 1** Two-dimensional examples for the eight base relations of RCC8*

disjunctions) of the relations, e. g., EC $\cup$ PO. If we want to express such indefinite information, we will use the following notation EC|PO(X, Y). Considering all possible unions, we get 2^8 different relations, among them the *universal relation* $\top$ which holds for all pairs of regions and the *impossible relation* $\bot$ that holds between no pairs of regions.

Using this set of relations, we could for example describe a particular spatial configuration of three regions X, Y, and Z as follows:

$$\Theta = \{\mathrm{TPP}(X, Y), \mathrm{DC|NTTP}(Y, Z)\}. \quad (1)$$

Now one can ask what additional relationships follow from Θ and whether it is possible to find regions that satisfy all the relationships simultaneously. For the given description Θ, one sees, for example, that we cannot add the formula NTPP(Z, X) and have still all relationships satisfied by some regions. On the other hand, we know that Z is either DC or NTPP^{-1} to X. In other words, we could add the statement DC|NTTP$^{-1}(Z, X)$ to Θ without changing anything. And this is, what constraint propagation is all about. We add derived statements about triples of objects as long as these statements are new and we have not derived an impossible relation.

The main research goal in this context is the analysis of the computational properties of deciding consistency of such qualitative descriptions of spatial configurations as well as related problems. This includes the analysis of the computational complexity and decidability as well as the design of efficient approximation and complete algorithms. For example, Renz and Nebel [30] identified all fragments of the RCC-8 calculus that permit tractable consistency problems and contain all eight base relations and devised an efficient algorithm for deciding consistency of the full calculus [31].

Interaction between Empirical and Synthetic Approaches to Cognition

Cognitive systems tend to be complex. As a consequence, only partial models can be constructed from empirical data that reliably reflect the structures and functions of the natural role models. On the other hand, running computational models must be complete on a given level of description to be executable. Thus, functional AI models must contain structures that are not based on empirical evidence but on the constructor's intuitions. A great advantage of synthetic systems is that all constructive elements and structures are known, at least in principle.

If we view natural and artificial agents as different implementations of a given cognitive functionality, we can actively explore differences in performance and adapt our implementations according to new insights. In this way, cognitive psychologists may extend their range of empirical studies to artificial agents and AI programs can help bridge the knowledge gap between structure and function of cognitive systems.

The neuroanatomist and cybernetician Braitenberg characterized the use of synthetic constructs for the exploration of natural systems by noting that a given performance always can be achieved by many different mechanisms [3]. He formulated the law of uphill analysis and downhill invention noting that it is easy to create little machines that produce surprising behavior by simple means and much more difficult to derive from the outside the internal structure from the observation of behavior. As a psychological consequence of this he noted that we tend to overestimate the complexity of a mechanism when we analyze it. His experiments suggest that by building and exploring synthetic structures on the basis of biological principles we may make the best progress towards understanding natural structures.

As mentioned at the beginning of this paper, Cognitive Science has research goals that differ widely from those of AI. And in fact, the complexity results mentioned in the previous section seem hardly of relevance for Cognitive Science. In human cognition asymptotic performance of an algorithm is not very interesting because humans can deal only with a very limited number of objects at the same time. Nevertheless, there are a number of questions that people in both areas find interesting and/or where a result of one area is relevant for the other area.

For example, when considering the calculus RCC8 mentioned above, the question arises whether the qualitative distinctions are on the right level. One could, for instance, also consider a system with only five relations which does not distinguish between the fact that two regions touch each other or not. In other words, the relation pairs EC and DC, TPP and NTPP, as well as TPP^{-1} and $NTPP^{-1}$ would be considered as indistinguishable. Knauff et al. [17] addressed this and other questions in order to determine the *conceptual adequacy* of the RCC8 calculus. As it turns out, RCC8 seems to be more cognitively adequate than the 5-relation system. Moreover, form and size appear as less prominent than the topological relations. Obviously, such results are important when one considers use of such a calculus in a human-computer interface. So, psychological experiments clearly have an impact on computer science research.

However, also the other way around, computer science results can have an impact on psychological research. For example, when Knauff et al. [16] tested a hypothesis concerning so-called *preferred relations*, they used an extensive formal analysis of Allen's relation system about the number of different models satisfying a given set of statements.

Furthermore, while the direct application of computational complexity theory to human cognition does not seem to make sense, the very idea of measuring the difficulty of cognitive tasks by counting the necessary operations of some computational model can well be applied. For instance, Ragni et al. [27] proposed a two-dimensional array on which a *spatial focus* operates, which is used to insert and inspect object proxies at particular places of this array. Counting the necessary operations for solving so-called *three-term series* tasks, it turned out that human subjects tend to minimize the number of such operations. While this observation seems to hold for a number of different tasks, sometimes direction-dependent effects can be observed, which results in a modified model that accounts for such effects [18].

Conclusion

Informatics and artificial intelligence provide the theory and practical tools to characterize and implement cognitive systems. These systems can be studied in much the same way as natural cognitive systems. Artificial cognitive systems as objects of research, however, have the great advantage that the mechanisms underlying their cognitive functions are known in detail; thus, principles of cognitive processing can be described in terms of fundamental information processing mechanisms. Typically, artificial cognitive systems differ in strengths and weaknesses from natural cognitive systems. While the strength of natural cognitive systems is in the availability and integration of a large variety of knowledge sources and experience, the strength of artificial systems is in the large and reliable working memory and its precision. This makes artificial cognitive systems excellent candidates to assist human cognizers and to complement their weaknesses. A major limitation of human cognition is the resource limitation especially with respect to working memory capacity and availability of factual information. Such limitations are partially compensated for by ingenious forms of abstraction, generalization, and cognitive off-loading as well as brilliant inference mechanisms; these allow humans to recover from situations in which most artificial systems are lost. Thus, a major challenge for cognitive systems research is to better understand natural forms of multifaceted knowledge organization and knowledge processing under resource limitation.

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